Limitations of Reluctance Networks to Model the Frequency-Dependent Leakage and Fringing Fluxes in Active Magnetic Thrust Bearings

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Leakage and Fringing Fluxes in Active Magnetic Thrust Bearings

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Application: Aerospace
Vibration Assisted Drilling of CFRP–Titanium–Composites

Example: KEBA LeviSpin

Conventional Drilling

Vibration Assisted Drilling

Radial: Maximization of bearing stiffness
- Low inductances and voltages, fast controllers
- Maximization of bandwidth of current control 4…5 kHz

Axial: 100 Hz-vibration with 10 μm precision

How do eddy currents affect the macroscopic leakage and fringing flux distribution?
**Active Magnetic Thrust Bearing**

**Working Principle**

**Combined active radial- and thrust bearing**

**Focus:** Thrust bearing with nonlaminated core

- Homopolar pm–bias flux $\Phi_b$
  - Constant for small displacements
  - Evenly distributed between both halves of the thrust bearing, back iron over radial bearing

- Homopolar control flux $\Phi_x$ with toroidal coil and control current $i$
  - Linear behavior achieved by differential principle:
    \[
    F = k_{\text{Geo}} \cdot \Phi_v \cdot \Phi_x
    \]
    \[F \sim \Phi_x\]

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**Figure:** Cross-section through active magnetic bearing
Active Thrust Bearing
Magnetic Field

- **Common assumption:** \( F \sim \Phi_x \sim i \) with \( \Phi_x = \frac{L_h i}{N} \)  
  - **Current control**

  **Condition:** Neglectable eddy currents (for radial bearings w/ laminated cores)

- **But:** Eddy Currents require additional magnetizing current
  - Magnetic skin effect caused by eddy currents lowers inductance for high frequencies:
    \[ L = L_{\text{eff}}(j\omega) \text{ and not } L = L_h = \text{const.} \]

- **Correct:** \( F \sim \Phi_x \sim i_{\mu h} \)  
  - **Flux control**

- **Problem:** \( \Phi_x \) is not known to the controller
  - **Fractional-order flux estimator**

\[ i = i_{\mu h} + i_{\mu e} \]

\[ \text{measurable force-related} \]

\[ (R_{Cu} \ll \omega L, \ t \ll T_h) \]
**Magnetic Bearing Control**

Current control with direct current measurement

- **Problem:** force-generating current $i_{\mu h}$ cannot be measured and is unknown to the controllers
- **Assumption:** $i_{\text{meas}} = i_{\mu h}$ is only valid in quasi-stationary state
- **Consequence:** Decrease of dynamic, bandwidth and stability of the position control

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**Diagram**

- Current controller
- Inverter
- Current plant

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No feedback of force generating current

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**Mathematical Expressions**

- $f^* \rightarrow 1 / k_i$
- $i_{\mu h} \rightarrow e_i$
- $u^* \rightarrow V_t$
- $1 / (1 + sT_t)$
- $u \rightarrow R_{Cu} + sL_h$
- $i_{\mu h} \rightarrow ki$
- $f$
Magnetic Bearing Control
Flux Control with Flux Estimation

- **Solution:** Determination of flux from measured coil current with flux estimator
- **Ansatz:** Modeling of eddy currents with frequency-dependent *effective inductance* $L_{\text{eff}}(j\omega)$
- **Optional:** Consideration of leakage and fringing fluxes (stationary)
- **Controller design:** small time constant $T_t$ + large time constant $T_h$ ➔ **Amplitude Optimum**

\[
\frac{T_h, f / N}{1 + s T_h, t} \frac{R_{Cu} + s L_{\text{eff}, t}(s)}{1}
\]
**Modeling – What is included?**

**Effective Inductance**

- **Eddy currents:** Solving of diffusion equation
  \[ B \int \frac{dA}{\Phi} N i_x/\Phi \sum R_i \frac{N^2}{R_{\text{eff}}} \Rightarrow L_{\text{eff}} \]
  - Sum of fractional transcendent systems \( f(\sqrt{j\omega}) \)

- **Saturation:** Coefficients of flux estimator cannot be implemented dependent on current load
  - Choice of pre-defined relative permeability \( \mu_r \) according to current load point.
  - ISMB17

- **Hysteresis:** Fractional All-Pass Filter
  - Frequency-dependent consideration for single load point
  - High effort, little benefit

- **Leakage and fringing fluxes:** reluctance network (RN)
  - Flux distribution heavily depends on magnetic skin effect
  - Hardly representable with RN over entire bandwidth
  - Stationary correction factors?

**Figure:** Magnetic circuit of thrust bearing separated into part reluctances
Determination of Full Reluctance Network (Static)

- Identification of all fluxes $\Phi$ by FEA for known mmf $\Theta$
- Analytical calculation of core reluctances (high accuracy)
- Computed of Leakage/fringing reluctances by solving of SLE

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Simplification of Full Reluctance Network (Static)

- 0) Static analysis of impact of leakage and fringing flux paths

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**Simplified Reluctance Network (Static)**

What is the purpose of the network?

**Determination of force:** \( L_{hf} \)

\[
\begin{align*}
\phi_t &= R_{ri} \phi_i \\
\phi_f &= 2 \cdot R_{g1} \phi_i \\
\phi_i &= 2 \cdot R_{g1} \phi_i
\end{align*}
\]

- Distinction between total and force-generating flux necessary ➤ flux divider

\[
L_{hf} = k_{uf} \cdot N^2 \frac{\Phi_f}{\Theta} = 1.043 \cdot L_h
\]

**Conclusions:**

- **Unexpected:** \( k_{uf} > 1 \) ➤ presence of fringing and leakage fluxes actually increases force!

- **Cause:** Fringing more dominant than leakage ➤ Most likely general rule for similar thrust bearings

- Both factors \( k_{ut} \) and \( k_{uf} \) differ significantly and should be considered separately!
Frequency-dependent flux density distribution

Figure: Flux density distribution in the core of the thrust bearing without permanent magnets for various frequencies, displayed flux density limit is kept constant at 1 T, displayed flux density limit is set to mean value inside air gap when exited with 1 A
Frequency-dependency of leakage and fringing fluxes
Influence of the magnetic skin effect

- Change of behavior above 100 Hz:
  - Leakage flux swirls crossing entire coil are not negligible anymore
  - Network simplifications barely possible
  - Not usable in real-time control systems
  - Leakage and fringing fluxes close to shaft disappear
  - ... close to the coil they are amplified
    - Complete shift of flux distribution
    - Little impact on total flux $\Phi_t$, as effects cancel each other out
    - Force-related flux $\Phi_f$ calculated by network basically becomes meaningless

- No known approach to consider the influence of magnetic skin effect on leakage and fringing flux distribution
Frequency–dependency of leakage and fringing fluxes
Error of Reluctance Network (RN) to FEA

\[
\Delta R_{\text{eff}} \text{ in dBH} = 10 \log_{10} \left( \frac{L'_{\text{eff}}}{R'_{\text{eff}}} \right)
\]

**Errors to FEA**

- **Absolute Amplitude Error**
  \[ \Delta R_{\text{eff}} \text{ in dBH}^{-1} \]

- **Relative Amplitude Error**
  \[ \left| \frac{\Delta R_{\text{eff}}}{R_{\text{fem}}} \right| \text{ in %} \]

- **Absolute Phase Error**
  \[ \Delta \phi \text{ in } ^\circ \]

**Force-related Inductance \( L'_{\text{eff}} = \frac{N^2}{R'_{\text{eff}}} \)**

- \( k_f = 1.055 \)
- \( k_f = 1.043 \)

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**Conclusion**

**Motivation:**
\[ F \sim \Phi_x \]

**Assumption:** Correct

\[ \Phi_x \sim L_h \cdot i_{\text{meas}} \]

**State of the art:**
\[ \Phi_x \sim |L_{\text{eff}}(j\omega)| \cdot i_{\text{meas}} \]

- **Position Control with underlying flux control and fractional-order flux estimator**

- **Aim:** Improvement of estimator by including leakage and fringing fluxes

- **Significant Impact:** 5% on \( L_{h_f} \) and 10% on \( L_{h_t} | L_{\text{eff}_t} \) (Accuracy)

**Literature suggests:** Reluctance Networks (RNs), but:
- Although accurate for static case, only insufficient consideration of magnetic skin effect
- Challenging to calculate (analytically), inefficient to implement in real-time
- Constant correction factors with higher accuracy over entire frequency range

- **Correction factors are simple, accurate and efficient!**

- **If FEA is available:** Reluctance Networks have no practical benefits! *Are they obsolete?*
Actually, it is not that complicated...

with the Riemann-Liouville definition:

\[ D^\alpha f(t) = \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dt} \int_0^t \frac{f(\tau)}{(t - \tau)^\alpha} \, d\tau \]

one determines the half derivation of \( t^2 \):

\[ _0D^{\frac{1}{2}}(t^2) = \frac{1}{\Gamma(1 - \frac{1}{2})} \frac{d}{dt} \int_0^t \frac{\tau^2}{(t - \tau)^{\frac{1}{2}}} \, d\tau = \frac{8t^3}{3\sqrt{\pi}} \]

Thank you for your attention!
Publications and Literature

[Ackermann1985] [Amrhein2016] [Bahr2016] [Baker1996] [Bertotti1998] [Bleuler1984] [Bleuler1994] [Bañuelos2017] [Cardelli2003] [Cauer1954] [Chassaing2008] [Dastjerdi2019] [Deschrijver2008] [Dirscherl2017] [Doyle1981] [Efe2011] [Elwakil2010] [Ernst2020] [Faiz2010] [Feeley1996] [Ferreira2017] [Flax1966] [Fleischer2011] [Fleischer2013] [Fleischer2017] [Gähler1998] [Ghasemi2014] [Grünwald1867] [Gustavsen1999] [Gustavsen2006] [Han2013] [Hecht2021] [Hemenway2021] [Herzog2009] [Horowitz2001] [Hutton1975] [Jaatinen2013] [Jackson1970] [Jackson1989] [Jalloul2013] [Keith1993] [Kessler1955] [Kessler1958] [Köhring2010] [Krasnoselskii1983] [Krishna2011] [Kucera1996] [Lammeraner1966] [Langholz1978] [Larsonneur1988] [Le2016] [León2014] [Levy1959] [Liebfried2018] [Liebfried2021] [Lino2017] [Luo2009] [Lutz2014] [Maione2006] [Maione2013] [Maslen2017] [Matignon1996] [Matsuda1993] [Mayorgyo1985] [McLachlan1955] [Meeker1996] [Milovanovic2015] [Mönch2015] [Monje2010] [Moon1961] [Müller2006] [Noda2005] [Nonami1994] [Nonami1996] [Novak2018] [Oldham1974] [Onyedi2020] [Oustaloup1983] [Oustaloup1995b] [Oustaloup1995a] [Oustaloup2000a] [Oustaloup2000a] [Oustaloup2000b] [Paszek1979] [Pecat2014] [Petrás2009] [Podlubny1999] [Preisach1935] [Rabinovich1992] [Radwan2009] [Retière1999] [Ribbenfjard2008] [Riemann1876] [Riu2003] [Rodriguez2007] [Roters1941] [Roy1967] [Rudolph2019] [Rüdenberg1953] [Sabatier2012] [Sanathanan1963] [Schröder2009] [Schuhmann2006] [Schuhmann2011] [Schweitzer1993] [Schweitzer2009] [Schweitzer2011] [Schwenk2012] [Scott1994] [Seifert2015] [Seifert2016] [Seifert2017b] [Seifert2017a] [Seifert2019a] [Seifert2019b] [Seifert2019c] [Seifert2021b] [Seifert2021a] [Shirreff2016] [Smith1996] [Spece2018] [Stiebler2005] [Stoll1974] [Sun2009] [Svaricek2016] [Swann2009] [Tepljakov2011] [Tepljakov2014] [Tepljakov2018] [Tepljakov2019] [Tepljakov2021] [Välimäki2016] [Vinagre2000] [Vinagre2003] [Vischer1988] [Weiner2018] [Weniger1990] [Whitlow2014] [Whitlow2016] [Whitlow2018] [Wiedemann1967] [Wong2008] [Yi1995] [Zif2013] [Zhong2014] [Zhong2015] [Zhou2016] [Zhu2004a] [Zhu2004b] [Zhu2005a] [Zhu2010] [Zingerli2010] [Zlatnik1990] [Zmood1987]